

# Electromagnetic Fields of Slowly Rotating Compact Magnetized Stars in Braneworld

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## Abstract

We study the structure of electromagnetic field of slowly rotating magnetized star in a Randall-Sundrum II type braneworld. The star is modeled as a sphere consisting of perfect highly magnetized fluid with infinite conductivity and frozen-in dipolar magnetic field. Maxwell's equations for the external magnetic field of the star in the braneworld are analytically solved in approximation of small distance from the surface of the star. We have also found numerical solution for the electric field outside the rotating magnetized neutron star in the braneworld in dependence on brane tension. The influence of brane tension on the electromagnetic energy losses of the rotating magnetized star is underlined. Obtained "brane" corrections are shown to be relevant and have non-negligible values. In comparison with astrophysical observations on pulsars spin-down data they may provide an evidence for the brane tension and, thus, serve as a test for the braneworld model of the Universe.

## 1 Introduction

The study of magnetic and electric fields around the compact objects is an important task for several reasons. First is that we obtain information about such stars through their observable characteristics, which are closely connected with electromagnetic fields inside and

outside the stars. Magnetic fields play an important role in the life history of majority astrophysical objects especially of compact relativistic stars which possess surface magnetic fields of  $10^{12}G$  and  $\sim 10^{14}G$  in the exceptional cases for magnetars. The strength of compact star's magnetic field is one of the main quantities determining their observability, for example as pulsars through magneto-dipolar radiation. Electric field surrounding the star determines energy losses from the star and therefore may be related with such observable parameters as period of pulsar and its time derivation. The second reason is that we may test various theories of gravitation through the study of compact objects for which general relativity effects are especially strong. Considering different metrics of space-time one may investigate the effect of the different phenomena on evolution and behavior of stellar interior and exterior magnetic fields. Then these models can be checked through comparison of theoretical results with observational data. The third reason may be seen in influence of stellar magnetic and electric field on different physical phenomena around the star, such as gravitational lensing and motion of test particles.

In the Newtonian framework the exterior electromagnetic fields of magnetized and rotating sphere are given in the classical paper of Deutsch (1955) and interior fields are studied by many authors, for example, in the paper of Ruffini & Treves (1973). In the general-relativistic approach the study of the magnetic field structure outside magnetized compact gravitational objects has been initiated by the pioneering work of Ginzburg & Ozernoy (1964) and have been further extended by number of authors (Anderson & Cohen (1970), Petterson (1974), Wasserman & Shapiro (1983), Muslimov & Harding (1997), Muslimov & Tsygan (1992), Rezzolla et.al. (2001a), Rezzolla et.al. (2001b), Kojima et.al. (2004)), while in some papers (Gupta et.al. (1998), Prasanna & Gupta (1997), Geppert et.al. (2000), Page et.al.

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(2000), Zanotti & Rezzolla (2002)) the work has been completed by considering magnetic fields interior relativistic star for the different models of stellar matter. General-relativistic treatment for the structure of external and internal stellar magnetic fields including numerical results has shown that the magnetic field is amplified by the monopolar part of gravitational field depending on the compactness of the relativistic star.

We are interested in study of stellar electric and magnetic fields in frames of recently popular model of the braneworld first proposed in the work of Randall & Sundrum (1999). According to this model our four-dimensional space-time is just a slice of five-dimensional bulk and only gravity is the force which can freely propagate between our space-time and bulk while other fields are confined to four-dimensional Universe. The review of braneworld models is given in the work of Maartens (2004) and some cosmological and astrophysical implications of the braneworld theories may be found in works Maartens (2000), Campos & Sopuerta (2001), Langlois (2001), Harko & Mak (2003), Gergely (2006), Kovacs & Gergely (2008), Majumdar & Mukherjee (2005). For astrophysical interests, static and spherically symmetric exterior vacuum solutions of the braneworld models were initially proposed by Dadhich et.al. (2000) which have the mathematical form of the Reissner-Nordström solution, in which a tidal Weyl parameter  $Q^*$  plays the role of the electric charge squared of the general relativistic solution. It should be noted that besides this solution was pioneering and there are many different vacuum braneworld solutions at the moment, this solution still stays interesting and actual and, for example, was recently applied to the solar system tests in the paper of Böhmer et.al. (2010).

Observational possibilities of testing the braneworld black hole models at an astrophysical scale have been intensively discussed in the literature during the last several years, for example, through the gravitational lensing, the motion of test particles, and the classical tests of general relativity (perihelion precession, deflection of light, and the radar echo delay) in the Solar System (see Böhmer et.al. (2008)). In the paper of Pun et.al. (2008) the energy flux, the emission spectrum, and accretion efficiency from the accretion disks around several classes of static and rotating braneworld black holes have been obtained. The complete set of analytical solutions of the geodesic equation of massive test particles in higher dimensional spacetimes which can be applied to braneworld models is provided in the recent paper of Hackmann et.al. (2008). The relativistic quantum interference effects in the spacetime of slowly rotating object in braneworld and phase shift effect of interfering particle in neutron

interferometer have been studied in the recent paper of Mamadjanov et.al. (2010). The influence of the tidal charge onto profiled spectral lines generated by radiating tori orbiting in the vicinity of a rotating black hole has been studied in the paper of Schee & Stuchlík (2009). Authors showed that with lowering the negative tidal charge of the black hole, the profiled line becomes flatter and wider, keeping their standard character with flux stronger at the blue edge of the profiled line. The role of the tidal charge in the orbital resonance model of quasiperiodic oscillations in black hole systems has been investigated in the paper of Stuchlík & Kotrlová (2009). The influence of the tidal charge parameter of the braneworld models on some optical phenomena in rotating black hole spacetimes has been extensively studied in the paper of Schee & Stuchlík (2009).

A braneworld corrections to the charged rotating black holes and to the perturbations in the electromagnetic potential around black holes are studied, for example, in works of Aliev & Gümrükçüoğlu (2005) and Abdujabbarov & Ahmedov (2010). Our preceding paper, Ahmedov & Fattoyev (2008), was devoted to the stellar magnetic field configurations of relativistic stars in dependence on brane tension. The present paper extends the paper of Ahmedov & Fattoyev (2008) to the case of rotating relativistic star. In our paper we will consider rotating spherically symmetric star in the braneworld endowed with strong magnetic fields. We assume that the star has dipolar magnetic field and the field energy is not strong enough to affect the spacetime geometry, so we consider the effects of the gravitational field of the star in the braneworld on the magnetic and electric field structure without feedback. The motion of test particles near black holes immersed in an asymptotically uniform magnetic field and some gravity surrounding structure, which provides the magnetic field has been intensively studied in the paper of Konoplya (2006). The author has calculated the binding energy for spinning particles on circular orbits. The bound states of the massive scalar field around a rotating black hole immersed in the asymptotically uniform magnetic field are considered in the paper of Konoplya (2007). The uniform magnetic field in the background of a five dimensional black hole has been extensively studied in the work of Aliev & Frolov (2004). In particular, authors presented exact expressions for two forms of an electromagnetic tensor and the electrostatic potential difference between the event horizon of a five dimensional black hole and the infinity.

The paper is organized as follows. In section 2 we present a set of Maxwell's equations in the space-time of spherically symmetric rotating relativistic compact star in the braneworld. Section 3 is devoted for solutions of

Maxwell's equations. In subsection 3.1 we consider the solution for "toy model" - monopolar structure of magnetic field of the star. This solution is not realistic but it can be used to obtain first estimates of the influence of brane tension on the electromagnetic field of the star. In subsection 3.2 we are looking for analytical solution of the Maxwell's equations for the exterior magnetic field of the star. We obtain approximate solution of the differential equation for magnetic field in the near vicinity of the surface of the star. In subsection 3.3 we get the differential equation for the electric field outside the star and solve them numerically. We show that both magnetic and electric fields will be essentially modified by five-dimensional gravity effects. In subsection 3.4 we investigate the astrophysical application of obtained result, namely, calculate energy losses from the slowly rotating magnetized neutron star in the braneworld. The last section is devoted to the conclusions of the research done.

Throughout, we use a space-like signature  $(-, +, +, +)$  and a system of units in which  $G = 1 = c$ . Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.

## 2 Maxwell Equations In a Spacetime of Slowly Rotating Spherical Star in the Braneworld

The metric for the space-time around rotating compact object in the braneworld may be written in coordinates  $u, r, \theta, \varphi$  as (see Pun et.al. (2008))

$$ds^2 = \left[ - (du + dr)^2 + dr^2 + \Sigma d\theta^2 + (r^2 + a^2) \sin^2 \theta d\varphi^2 + 2a \sin^2 \theta dr d\varphi + G(du - a \sin^2 \theta d\varphi)^2 \right], \quad (1)$$

where  $G = (2Mr - Q^*)/\Sigma$ ,  $\Sigma = r^2 + a^2 \cos^2 \theta$ ,  $Q^*$  is the bulk tidal charge,  $M$  is the mass of the star, parameter  $a$  is related to the angular momentum of the star. In this form the metric was presented in the paper of Aliev & Gümrükçüoğlu (2005) and obtained in assumption that the axisymmetric and stationary braneworld metric has the Kerr-Schild form and can be expressed in the form of its linear approximation around the flat metric  $ds^2 = (ds^2)_{flat} + H(l_\mu dx^\mu)^2$ , where  $l_\mu$  is a null, geodesic vector field in both the flat and full metrics, and  $H$  is an arbitrary scalar function.

Applying the Boyer-Lindquist transformation  $du = dt - (r^2 + a^2)dr/\Delta$ ,  $d\varphi = d\phi - adr/\Delta$  with  $\Delta = r^2 + a^2 - 2Mr + Q^*$  and assuming parameter  $a$  to be

small and assuming parameter  $a$  to be small one and assuming parameter  $a$  to be small one can obtain the exterior metric for slowly rotating neutron star in the braneworld in the following form

$$ds^2 = -A^2 dt^2 + H^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - 2\tilde{\omega}(r)r^2 \sin^2 \theta dt d\phi, \quad (2)$$

where

$$A^2(r) \equiv \left( 1 - \frac{2M}{r} + \frac{Q^*}{r^2} \right) = H^{-2}(r), \quad r > R, \quad (3)$$

is the Reissner-Nordström-type exact solution for the metric outside the star,  $\tilde{\omega}(r) = \omega(1 - Q^*/2rM)$ ,  $\omega = 2Ma/r^3$  is the angular velocity of the dragging of inertial frames.

The general form of the first pair of general relativistic Maxwell equations is given by

$$3!F_{[\alpha\beta,\gamma]} = 2(F_{\alpha\beta,\gamma} + F_{\gamma\alpha,\beta} + F_{\beta\gamma,\alpha}) = 0, \quad (4)$$

where  $F_{\alpha\beta}$  is the electromagnetic field tensor.

The covariant components of the electromagnetic tensor are explicitly expressed in terms of electric and magnetic field components as

$$F_{\alpha\beta} \equiv 2u_{[\alpha}E_{\beta]} + \eta_{\alpha\beta\gamma\delta}u^\gamma B^\delta, \quad (5)$$

where  $u^\alpha$  is the four-velocity of observer,  $T_{[\alpha\beta]} \equiv \frac{1}{2}(T_{\alpha\beta} - T_{\beta\alpha})$  and  $\eta_{\alpha\beta\gamma\delta}$  is the pseudo-tensorial expression for the Levi-Civita symbol  $\epsilon_{\alpha\beta\gamma\delta}$

$$\eta^{\alpha\beta\gamma\delta} = -\frac{1}{\sqrt{-g}}\epsilon_{\alpha\beta\gamma\delta}, \quad \eta_{\alpha\beta\gamma\delta} = \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta}, \quad (6)$$

with  $g \equiv \det|g_{\alpha\beta}| = -A^2 H^2 r^4 \sin^2 \theta$ .

The general form of the second pair of Maxwell equations is given by

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^\alpha \quad (7)$$

where the four-current  $J^\alpha$  is a sum of convection and conduction currents

$$J^\alpha = \rho_e w^\alpha + j^\alpha, \quad j^\alpha w_\alpha \equiv 0, \quad j_\alpha = \sigma F_{\alpha\beta} w^\beta, \quad (8)$$

where  $\sigma$  is the electrical conductivity and  $w^\alpha$  is four-velocity of conducting medium as a whole.

The four-velocity of the 'zero angular momentum observer' (ZAMO) in the metric 2 looks like

$$(u^\alpha)_{\text{obs}} \equiv A^{-1} \begin{pmatrix} 1, 0, 0, \tilde{\omega} \end{pmatrix}; \quad (u_\alpha)_{\text{obs}} \equiv A \begin{pmatrix} -1, 0, 0, 0 \end{pmatrix}. \quad (9)$$

Using the main equations (4), (7) together with (5), (6) and (9) one can obtain Maxwell equations for slowly rotating neutron star in the braneworld for the orthonormal reference frame as

$$\sin \theta (r^2 B^{\hat{r}})_{,r} + Hr \left( \sin \theta B^{\hat{\theta}} \right)_{,\theta} + Hr B_{,\phi}^{\hat{\phi}} = 0 \quad (10)$$

$$(r \sin \theta) \frac{\partial B^{\hat{r}}}{\partial t} = A \left[ E_{,\phi}^{\hat{\theta}} - \left( \sin \theta E^{\hat{\phi}} \right)_{,\theta} \right] - (\tilde{\omega} r \sin \theta) B_{,\phi}^{\hat{r}}, \quad (11)$$

$$(Hr \sin \theta) \frac{\partial B^{\hat{\theta}}}{\partial t} = -A H E_{,\phi}^{\hat{r}} + \sin \theta \left( r A E^{\hat{\phi}} \right)_{,r} - (H \tilde{\omega} r \sin \theta) B_{,\phi}^{\hat{\theta}}, \quad (12)$$

$$(Hr) \frac{\partial B^{\hat{\phi}}}{\partial t} = - \left( r A E^{\hat{\theta}} \right)_{,r} + A H E_{,\theta}^{\hat{r}} + \sin \theta (\tilde{\omega} r^2 B^{\hat{r}})_{,r} + H \tilde{\omega} r (\sin \theta B^{\hat{\theta}})_{,\theta} \quad (13)$$

and

$$\sin \theta (r^2 E^{\hat{r}})_{,r} + Hr \left( \sin \theta E^{\hat{\theta}} \right)_{,\theta} + Hr E_{,\phi}^{\hat{\phi}} = 4\pi H r^2 \sin \theta J^{\hat{t}}, \quad (14)$$

$$A \left[ \left( \sin \theta B^{\hat{\phi}} \right)_{,\theta} - B_{,\phi}^{\hat{\theta}} \right] - (\tilde{\omega} r \sin \theta) E_{,\phi}^{\hat{r}} = (r \sin \theta) \frac{\partial E^{\hat{r}}}{\partial t} + 4\pi A r \sin \theta J^{\hat{r}}, \quad (15)$$

$$A H B_{,\phi}^{\hat{r}} - \sin \theta \left( r A B^{\hat{\phi}} \right)_{,r} - (H \tilde{\omega} r \sin \theta) E_{,\phi}^{\hat{\theta}} = (Hr \sin \theta) \frac{\partial E^{\hat{\theta}}}{\partial t} + 4\pi A H r \sin \theta J^{\hat{\theta}}, \quad (16)$$

$$\left( A r B^{\hat{\theta}} \right)_{,r} - A H B_{,\theta}^{\hat{r}} + \sin \theta (\tilde{\omega} r^2 E^{\hat{r}})_{,r} + H \tilde{\omega} r (\sin \theta E^{\hat{\theta}})_{,\theta} = (Hr) \frac{\partial E^{\hat{\phi}}}{\partial t} + 4\pi A H r J^{\hat{\phi}} + 4\pi H \tilde{\omega} r^2 \sin \theta J^{\hat{t}}. \quad (17)$$

### 3 Stationary Solutions to Maxwell Equations

We will look for stationary solutions of the Maxwell equation, i.e. for solutions in which we assume that the magnetic moment of the star does not vary in time as a result of the infinite conductivity of the stellar interior.

#### 3.1 A Special Monopolar Configuration for Magnetic Field of Slowly Rotating Star in Braneworld

First we consider the following magnetic field configuration as a toy model

$$B^{\hat{r}} = B^{\hat{r}}(r) \neq 0, \quad B^{\hat{\theta}} = 0. \quad (18)$$

Although this form of magnetic field can not be considered realistic, we will show that this toy model can be used to obtain the first estimates of the influence of brane tension on the electromagnetic field of the star. For this case, in the linear approximation in the Lense-Thirring frequency  $\omega$  Maxwell equations (10) and (17) reduce to

$$(r^2 B^{\hat{r}})_{,r} = 0, \quad B_{,\theta}^{\hat{r}} = 0. \quad (19)$$

The solution admitted by this equation is

$$B^{\hat{r}} = \frac{\tilde{\mu}}{r^2}, \quad (20)$$

where  $\tilde{\mu}$  is the integration constant being responsible for the source of the monopolar magnetic field.

For the chosen configuration of the magnetic field due to the infinite conductivity of the stellar matter one can easily find the electric field as

$$E^{\hat{\theta}} \neq 0, \quad E^{\hat{r}} = E^{\hat{\phi}} = 0. \quad (21)$$

Then, analytical solution of the Maxwell equation

$$(r A E^{\hat{\theta}})_{,r} - \tilde{\mu} \sin \theta (\tilde{\omega})_{,r} = 0 \quad (22)$$

has the following form

$$E^{\hat{\theta}} = \frac{\tilde{\omega} - C}{r A} \tilde{\mu} \sin \theta, \quad (23)$$

where  $C$  is the integration constant (see also Abdujabbarov et.al. (2008) for the case of slowly rotating NUT star). To find  $C$  we match (23) with the known inner solution for the electric field of the rotating star in the Newtonian spacetime. For the star rotating with the angular frequency  $\Omega$  we have (see Rezzolla & Ahmedov (2004))

$$E_{in}^{\hat{\theta}} = - \frac{\Omega r \sin \theta B_{in}^{\hat{r}}}{N}. \quad (24)$$

Remember now that the radial component of the magnetic field and tangential components of the electric field are continuous at the surface of the star and obtain (23) as

$$E^{\hat{\theta}} = \frac{\tilde{\omega} - \Omega}{r A} \tilde{\mu} \sin \theta. \quad (25)$$

In the Fig. 1 radial dependence of monopolar  $E/E_{q=0}$  is shown for the several values of parameter  $q = Q^*/M^2$  (we assume compactness parameter  $\varepsilon = M/R = 1/3$ ). One can see when the star has monopolar magnetic field the electric field is noticeably diminished due to the presence of the brane tension.

### 3.2 Stationary Solutions to Maxwell Equations for Dipolar Magnetic Field of Slowly Rotating Magnetized Highly Conducting Star in Braneworld

Now we look for solution of the Maxwell equations (10)–(13) and (14)–(17) assuming the realistic configurations of the magnetic field of the star i.e. the dipolar one.

We look for separable solutions of Maxwell equations in the form

$$\begin{aligned} B^{\hat{r}}(r, \theta) &= F(r) \cos \theta, \quad B^{\hat{\theta}}(r, \theta) = G(r) \sin \theta, \\ B^{\hat{\phi}}(r, \theta) &= 0, \end{aligned} \quad (26)$$

assuming that magnetic field of the star is dipolar and functions  $F(r)$  and  $G(r)$  will account for the relativistic corrections due to a curved background spacetime.

Maxwell equations (10), (15)–(17) with the ansatz (26), yield the following set of equations

$$(r^2 F)_{,r} + 2HrG = 0, \quad (27)$$

$$(rAG)_{,r} + F = 0. \quad (28)$$

The exterior solution for the magnetic field is simplified by the knowledge of explicit analytic expressions for the metric functions  $A$  and  $H$ . In particular, after defining  $A = H^{-1} = (1 - 2M/r + Q^*/r^2)^{1/2}$ , the system (27)–(28) can be written as a single, second-order ordinary differential equation for the unknown function  $F$

$$\frac{d}{dr} \left[ \left( 1 - \frac{2M}{r} + \frac{Q^*}{r^2} \right) \frac{d}{dr} (r^2 F) \right] - 2F = 0. \quad (29)$$

The analytical solution of equation (29) exists in the literature when the parameter  $Q^* = 0$  (see, for example, Rezzolla et.al. (2001a), Rezzolla et.al. (2001b)). In the case  $Q^* \neq 0$  the equation is more complicated but it allows analytical solution under simplification near the surface of the star. First we rewrite equation (29) as

$$\begin{aligned} r^2 \left( 1 - \frac{2M}{r} + \frac{Q^*}{r^2} \right) \frac{d^2 F}{dr^2} \\ + 4r \left( 1 - \frac{3M}{2r} + \frac{Q^*}{2r^2} \right) \frac{dF}{dr} - \frac{2Q^*}{r^2} F = 0 \end{aligned} \quad (30)$$

and introduce the new variable  $z$  so that  $r = R + \delta = R(1 + z)$ ,  $z = \delta/R$ . Using also compactness parameter  $\varepsilon = M/R$  and introducing  $q = Q^*/M^2$  we get

$$\begin{aligned} [(1+z)^4 - 2\varepsilon(1+z)^3 + q\varepsilon^2(1+z)^2] \frac{d^2 F}{dz^2} \\ + [4(1+z)^3 - 6\varepsilon(1+z)^2 + 2q\varepsilon^2(1+z)] \frac{dF}{dz} \\ - 2q\varepsilon^2 F = 0. \end{aligned} \quad (31)$$

Now one may consider the region just above the surface of the star, where  $z \ll 1$ , expand the coefficients standing before the derivatives into series and leave only terms of first order of  $z$  and get

$$\begin{aligned} [(2q\varepsilon^2 - 6\varepsilon + 4)z + (q\varepsilon^2 - 2\varepsilon + 1)] \frac{d^2 F}{dz^2} \\ + [(2q\varepsilon^2 - 12\varepsilon + 12)z + (2q\varepsilon^2 - 6\varepsilon + 4)] \frac{dF}{dz} \\ - 2q\varepsilon^2 F = 0. \end{aligned} \quad (32)$$

Obtained equation is the particular case of equation (see Kamke (1959))

$$\begin{aligned} (a_2 x + b_2) f'' + (a_1 x + b_1) f' + (a_0 x + b_0) f = 0, \\ |a_2| + |b_2| > 0, \end{aligned} \quad (33)$$

which in the case when  $a_2 \neq 0$  with the help of substitutions  $f(x) = e^{kx} \eta(\xi)$ ,  $a_2 \xi = a_2 x + b_2$ , where  $k$  is the root of the square equation  $a_2 k^2 + a_1 k + a_0 = 0$  may be rewritten in the following form

$$\begin{aligned} a_2 \xi \eta'' + \left[ (2sa_2 + a_1) \xi + \frac{a_2 b_1 - a_1 b_2}{a_2} \right] \eta' \\ + \left( \frac{a_2 b_0 - a_0 b_2}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2} s \right) \eta = 0. \end{aligned} \quad (34)$$

In the case of our equation (32)  $a_0 = 0$ , so  $s = 0$  will be one of the roots of square equation. This will make the substitutions to be  $F(z) = \eta(\xi)$ ,  $\xi = z + b_2/a_2$ , where

$$\begin{aligned} a_2 &= (2q\varepsilon^2 - 6\varepsilon + 4), & b_2 &= (q\varepsilon^2 - 2\varepsilon + 1), \\ a_1 &= (2q\varepsilon^2 - 12\varepsilon + 12), & b_1 &= (2q\varepsilon^2 - 6\varepsilon + 4), \\ b_0 &= -2q\varepsilon^2, \end{aligned} \quad (35)$$

so equation (32) may be rewritten as

$$\xi \eta'' + (a\xi + b) \eta' + (c\xi + d) \eta = 0 \quad (36)$$



with

$$\begin{aligned} a &= \frac{\varepsilon^2 q - 6\varepsilon + 6}{\varepsilon^2 q - 3\varepsilon + 2}, \\ b &= \frac{\varepsilon^4 q^2 - 4\varepsilon^3 q + \varepsilon^2(6 + q) - 6\varepsilon + 2}{2(\varepsilon^2 q - 3\varepsilon + 2)^2}, \\ c &= 0, \quad d = -\frac{\varepsilon^2 q}{\varepsilon^2 q - 3\varepsilon + 2}. \end{aligned} \quad (37)$$

Equation (36) belongs to type of degenerate hypergeometric equation. Solution of this equation have the following sign (Kamke (1959))

$$\begin{aligned} \eta &= \xi^{-\frac{b}{2}} e^{-\frac{a\xi}{2}} \left[ C_1 (a\xi)^{\frac{b}{2}} e^{-\frac{a\xi}{2}} {}_1F_1 \left( b - \frac{d}{a}, b, a\xi \right) \right. \\ &\quad \left. + C_2 (a\xi)^{1-\frac{b}{2}} e^{-\frac{a\xi}{2}} {}_1F_1 \left( 1 - \frac{d}{a}, 2 - b, a\xi \right) \right], \end{aligned} \quad (38)$$

where the constants  $C_1$  and  $C_2$  can be determined from the boundary conditions and definition

$${}_1F_1(l, m, x) = 1 + \sum_{n=1}^{\infty} \frac{l(l+1)\dots(l+n-1)x^n}{m(m+1)\dots(m+n-1)n!}. \quad (39)$$

Turning back to  $F(z)$  from  $\eta(\xi)$ , one can get the radial eigenfunction of magnetic field outside the slowly rotating star in the braneworld as

$$\begin{aligned} F(z) &= (z+s)^{-\frac{b}{2}} e^{-a(z+s)} \\ &\times \left\{ C_1 [a(z+s)]^{\frac{b}{2}} {}_1F_1 \left( b - \frac{d}{a}, b, a(z+s) \right) \right. \\ &\quad \left. + C_2 [a(z+s)]^{1-\frac{b}{2}} \right. \\ &\quad \left. \times {}_1F_1 \left( 1 - \frac{d}{a}, 2 - b, a(z+s) \right) \right\}, \end{aligned} \quad (40)$$

where

$$s = \frac{\varepsilon^2 q - 2\varepsilon + 1}{2\varepsilon^2 q - 6\varepsilon + 4}. \quad (41)$$

The integration constants  $C_1$  and  $C_2$  may be found with the help of exact solution for the case  $q = 0$ , which has the following form (see e.g. Rezzolla et.al. (2001a))

$$F_{exact}(r) = -\frac{3}{4M^3} \left[ \ln N^2 + \frac{2M}{r} \left( 1 + \frac{M}{r} \right) \right] \mu \quad (42)$$

and using variable  $z$  one can obtain it as

$$\begin{aligned} F_{exact}(z) &= -\frac{3}{4M^3} \left[ \ln \left( 1 - \frac{2}{z+\varepsilon} \right) \right. \\ &\quad \left. + \frac{2}{z+\varepsilon} \left( 1 + \frac{1}{z+\varepsilon} \right) \right] \mu, \end{aligned} \quad (43)$$

where  $\mu$  is dipolar moment.

Using conditions

$$\begin{aligned} F(z)_{q=0}|_{z=0} &= F_{exact}(z)|_{z=0}, \\ \frac{dF(z)}{dz}|_{z=0} &= \frac{dF_{exact}(z)}{dz}|_{z=0} \end{aligned} \quad (44)$$

and assuming  $\varepsilon = 1/3$  we numerically found  $C_1 \approx 0.28\mu/M^3$ ,  $C_2 \approx -0.25\mu/M^3$ .

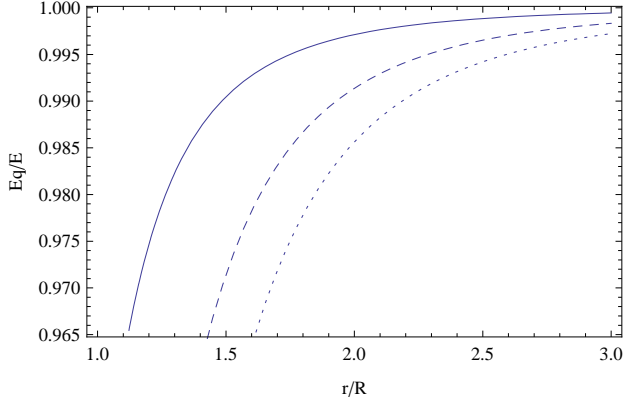
The function  $G(z)$  may be found through the relation

$$G(r) = -\frac{1}{2r} A(r) (r^2 F)_{,r}. \quad (45)$$

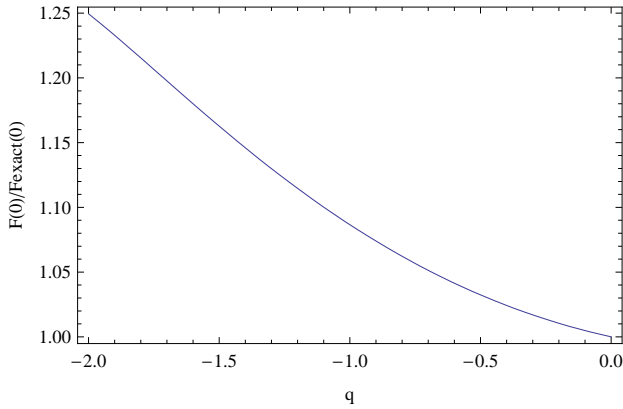
$F(z)$  and  $G(z)$  as functions of  $q$  are presented in the figures 2 and 3, correspondingly. Functions are taken at the surface of the star and normed on its exact values for the case  $Q^* = 0$ . It is seen from the graphs that the value of the surface magnetic field is noticeably modified due to the presence of brane tension  $Q^*$ , especially the radial component of  $B$ . The radial component  $B^{\hat{r}}(R, \theta)$  increases with the growth of  $|q|$  while the angular component  $B^{\hat{\theta}}(R, \theta)$  decreases. It should be noted that the surface value of magnetic field is of great importance and has strong influence on conditions of emission generation and energy losses from the rotating magnetized star.

### 3.3 Solution for Electric Field of Slowly Rotating Magnetized Highly Conducting Star in Braneworld

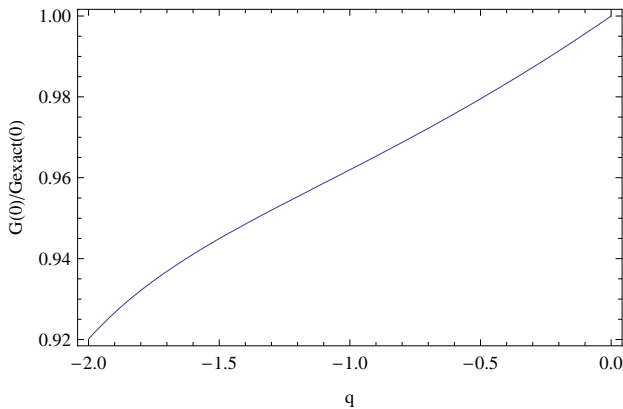
The search of solution for the electric field is more complicated with compare to that for the magnetic field. To simplify our task we will use the exact solution for magnetic field, obtained in the work of Rezzolla et.al. (2001a) (for the case  $Q^* = 0$ ), and, therefore, consider first order corrections by angular velocity and brane tension. With the help of exact expressions for the stationary magnetic field external to a misaligned magne-



**Fig. 1** Radial dependence of the ratio of electric field to that when  $q = 0$  for the several values of  $q = Q^*/M^2$ : solid line corresponds to  $q = -1$ , dashed to  $q = -3$ , dotted to  $q = -5$ .



**Fig. 2**  $F(z)|_{z=0}/F_{exact}|_{z=0}$  as a function of  $q = Q^*/M^2$ .



**Fig. 3**  $G(z)|_{z=0}/G_{exact}|_{z=0}$  as a function of  $q = Q^*/M^2$ .

tized relativistic star:

$$B^{\hat{r}} = -\frac{3}{4M^3} \left[ \ln N^2 + \frac{2M}{r} \left( 1 + \frac{M}{r} \right) \right] \times (\cos \chi \cos \theta + \sin \chi \sin \theta \cos \lambda) \mu, \quad (46)$$

$$B^{\hat{\theta}} = \frac{3N}{4M^2 r} \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] \times (\cos \chi \sin \theta - \sin \chi \cos \theta \cos \lambda) \mu, \quad (47)$$

$$B^{\hat{\phi}} = \frac{3N}{4M^2 r} \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] (\sin \chi \sin \lambda) \mu \quad (48)$$

one can obtain Maxwell equations for the electric field around misaligned star in the braneworld as

$$A \left[ (\sin \theta E^{\hat{\phi}})_{,\theta} - E^{\hat{\theta}}_{,\phi} \right] = \frac{3\tilde{\omega} r}{4M^3} \mu \left[ \ln N^2 + \frac{2M}{r} \left( 1 + \frac{M}{r} \right) \right] \sin \chi \sin^2 \theta \sin \lambda, \quad (49)$$

$$E^{\hat{r}}_{,\phi} - \sin \theta (r A E^{\hat{\phi}})_{,r} = \frac{N}{A} \frac{3\tilde{\omega}}{4M^2} \mu \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] \sin \chi \sin \theta \cos \theta \sin \lambda, \quad (50)$$

$$(r A E^{\hat{\theta}})_{,r} - E^{\hat{r}}_{,\theta} = \frac{3\tilde{\omega}}{2M^2} \left( \frac{N}{A} - 1 \right) \mu \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] \sin \theta (\cos \theta \cos \chi + \sin \chi \cos \lambda \sin \theta) + \frac{N}{A} \frac{3\tilde{\omega}}{4M^2} \mu \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] \sin \chi \cos \lambda + \frac{3\tilde{\omega}}{4M^3} \mu \frac{\left( 1 - \frac{2Q^*}{3rM} \right)}{\left( 1 - \frac{Q^*}{2rM} \right)} \left[ \ln N^2 + \frac{2M}{r} \left( 1 + \frac{M}{r} \right) \right] \times \sin \theta (\cos \theta \cos \chi + \sin \chi \cos \lambda \sin \theta), \quad (51)$$

$$A \sin \theta (r^2 E^{\hat{r}})_{,r} + r (\sin \theta E^{\hat{\theta}})_{,\theta} + r E^{\hat{\phi}}_{,\phi} = 0. \quad (52)$$

The form of the external electromagnetic field around a misaligned rotating magnetized sphere was found in the paper of Deutsch (1955). Taking into account these solutions we look for the simplest solutions for the equations (49) in the following form

$$E^{\hat{r}} = (f_1 + f_3) \cos \chi (3 \cos^2 \theta - 1) + (g_1 + g_3) 3 \sin \chi \cos \lambda \sin \theta \cos \theta, \quad (53)$$

$$E^{\hat{\theta}} = (f_2 + f_4) \cos \chi \sin \theta \cos \theta + (g_2 + g_4) \sin \chi \cos \lambda - (g_5 + g_6) (\cos^2 \theta - \sin^2 \theta) \sin \chi \cos \lambda, \quad (54)$$

$$E^{\hat{\phi}} = [g_5 + g_6 - (g_2 + g_4)] \sin \chi \cos \theta \sin \lambda, \quad (55)$$

where the eigenfunctions  $f_1 - f_4$ ,  $g_1 - g_6$  have the radial dependence only. Using Maxwell equations (49) one can find the following set of linear differential equations for these functions:

$$A(r^2 f_1)_{,r} + r f_2 = 0, \quad (56)$$

$$(r A f_2)_{,r} + 6 f_1 = 0, \quad (57)$$

$$A(r^2 f_3)_{,r} + r f_4 = 0, \quad (58)$$

$$\begin{aligned} (r A f_4)_{,r} + 6 f_3 - \frac{9\tilde{\omega}r}{4M^3}\mu \frac{\left(1 - \frac{2Q^*}{3rM}\right)}{\left(1 - \frac{Q^*}{2rM}\right)} \\ \times \left[ \ln N^2 + \frac{2M}{r} \left(1 + \frac{M}{r}\right) \right] \\ - \frac{6\tilde{\omega}}{4M^2}\mu \left( \frac{N}{A} - 1 \right) \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] = 0 \end{aligned} \quad (59)$$

$$A(r^2 g_1)_{,r} + 2r g_5 = 0, \quad (60)$$

$$(r A f_5)_{,r} + 3 g_1 = 0, \quad (61)$$

$$A(r^2 g_3)_{,r} + 2r g_6 = 0, \quad (62)$$

$$\begin{aligned} (r A g_6)_{,r} + 3 g_3 - \frac{9\tilde{\omega}r}{8M^3}\mu \frac{\left(1 - \frac{2Q^*}{3rM}\right)}{\left(1 - \frac{Q^*}{2rM}\right)} \\ \times \left[ \ln N^2 + \frac{2M}{r} \left(1 + \frac{M}{r}\right) \right] \\ - \frac{6\tilde{\omega}}{8M^2}\mu \left( \frac{N}{A} - 1 \right) \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] = 0 \end{aligned} \quad (63)$$

Combining properly these equations one can obtain the following differential equations of second order for unknown functions  $f_1$  and  $f_3$  as

$$\frac{d}{dr} \left[ \left( 1 - \frac{2M}{r} + \frac{Q^*}{r^2} \right) \frac{d}{dr} (r^2 f_1) \right] - 6 f_1 = 0, \quad (64)$$

$$\frac{d}{dr} \left[ \left( 1 - \frac{2M}{r} + \frac{Q^*}{r^2} \right) \frac{d}{dr} (r^2 f_3) \right] - 6 f_3 \quad (65)$$

$$\begin{aligned} + \frac{9\tilde{\omega}r}{4M^3}\mu \frac{\left(1 - \frac{2Q^*}{3rM}\right)}{\left(1 - \frac{Q^*}{2rM}\right)} \left[ \ln N^2 + \frac{2M}{r} \left(1 + \frac{M}{r}\right) \right] \\ + \frac{6\tilde{\omega}}{4M^2}\mu \left( \frac{N}{A} - 1 \right) \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] = 0 \end{aligned} \quad (66)$$

From the system of equations (56) one can notice that functions  $f$  and  $g$  are connected with the following relations

$$g_1 = f_1, \quad g_3 = f_3, \quad g_5 = \frac{f_2}{2}, \quad g_6 = \frac{f_4}{2}. \quad (67)$$

Functions  $g_2$  and  $g_4$  can be found directly from the system of Maxwell equations (49), using (53), as

$$g_2 = \frac{3\Omega r}{8M^3 c A} \left[ \ln N^2 + \frac{2M}{r} \left(1 + \frac{M}{r}\right) \right] \mu, \quad (68)$$

$$g_4 = -\frac{\tilde{\omega}}{\Omega} g_2 = -\frac{3\tilde{\omega}r}{8M^3 c A} \left[ \ln N^2 + \frac{2M}{r} \left(1 + \frac{M}{r}\right) \right] \mu. \quad (69)$$

We are looking for numerical solutions of the second order ordinary differential equation (64) with the help of the Runge-Kutta fifth order method. To do this we assume the solution to be asymptotically Newtonian, namely

$$E_{Newt}^{\hat{r}} = -\frac{\mu \Omega R^2}{c r^4}, \quad (70)$$

since in the  $r \rightarrow \infty$  limit  $Q^*/r^2$  and  $M/r$  are negligibly small and do not give any contribution to the magnetic field. As initial conditions to solve ODE we have taken the values of the Newtonian expression and its derivation on a some large radii. With such a prescription, the equation is integrated inwards up to the surface of the relativistic star. Corresponding graphs are presented in the figure 4 for the different values of brane tension  $q = Q^*/M^2$ .

As it is seen from the figure, the modification of electric field for the different values of  $q$  has the magnitude of the order of tens percents of the standard value for the case when  $q = 0$ .

### 3.4 Astrophysical Consequences

Assume that the oblique rotating magnetized star is observed as radio pulsar through magnetic dipole radiation. Then the luminosity of the relativistic star in the case of a purely dipolar radiation, and the power radiated in the form of dipolar electromagnetic radiation, is given by Rezzolla & Ahmedov (2004)

$$L_{em} = \frac{\Omega^4 R^6 B_R^2}{6c^3} \sin^2 \chi, \quad (71)$$

where subscript  $R$  denotes the value at  $r = R$ .

Considering slowly rotating magnetized neutron star in the braneworld model one can see that the general relativistic braneworld corrections emerging in expression (71) will be partly due to the magnetic field amplification at the stellar surface and partly to the increase in the effective rotational angular velocity produced by the gravitational redshift as  $\Omega_Q^* = \Omega/A_R$ .

The presence of a braneworld tension has the effect of enhancing the rate of energy loss through dipolar electromagnetic radiation by an amount which can be



easily estimated to be

$$\frac{L_{em\ q \neq 0}}{L_{em\ q=0}} = \left( \frac{F_R}{F_{exact\ R}} \right)^2 \left( \frac{N_R}{A_R} \right)^4, \quad (72)$$

whose dependence from  $q$  is shown in Fig. (5) and which may reach significant values depending on the value of the brane tension parameter.

Noting that the energy losses of the pulsar may be related with the frequency of its rotation and the derivative of this frequency by time as

$$L_{em} = -\tilde{I}(\Omega \dot{\Omega}) \quad (73)$$

where  $\tilde{I}$  is the general relativistic moment of inertia of the star (see, for example Morozova et.al. (2008)),

$$\tilde{I} \equiv \int d^3\mathbf{x} \frac{\sqrt{\gamma}}{N} \rho r^2 \sin^2 \theta, \quad (74)$$

where  $\rho(r)$  is the total energy density,  $\gamma$  is the determinant of the three metric of slowly rotating star and  $d^3\mathbf{x}$  is the coordinate volume element, one may find the following relation for the time derivatives of rotation period of the pulsar

$$\frac{\dot{P}_{q \neq 0}}{\dot{P}_{q=0}} = \frac{\tilde{I}}{\tilde{I}_q} \frac{N_R}{A_R} \left( \frac{F_R}{F_{exact\ R}} \right)^2, \quad (75)$$

where

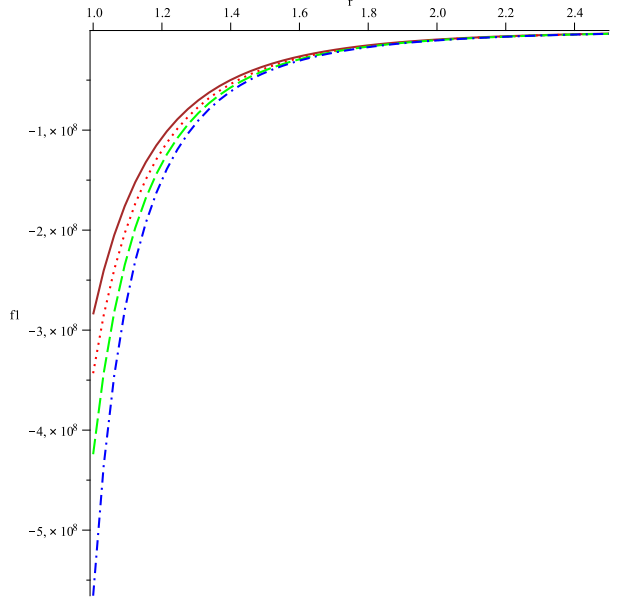
$$\tilde{I}_q \equiv \int d^3\mathbf{x} \frac{\sqrt{\gamma_q}}{A} \rho r^2 \sin^2 \theta, \quad (76)$$

and  $\gamma_q$  is the determinant of the three metric of slowly rotating neutron star in the braneworld determined by equation (2).

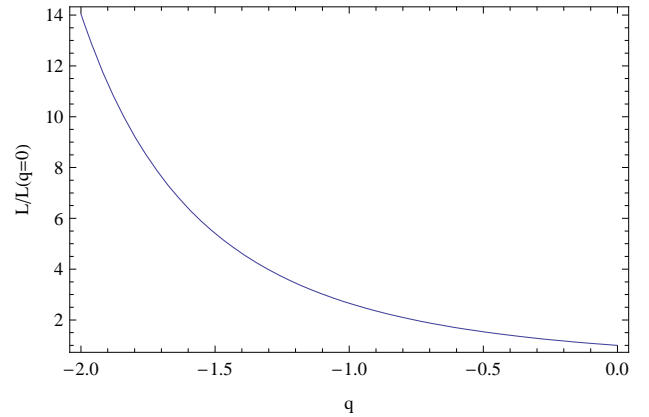
Expression (71) could be used to investigate the rotational evolution of magnetized neutron stars with predominant dipolar magnetic field anchored in the crust which converting its rotational energy into electromagnetic radiation. It should be also noted, that detailed investigation of general relativistic effects for Schwarzschild stars has already been performed by Page et.al. (2000), who have paid special attention to the general relativistic corrections that need to be included for a correct modeling of the thermal evolution but also of the magnetic and rotational evolution.

#### 4 Conclusions

In the present paper we considered modifications of electromagnetic field of a rotating magnetized neutron stars in the braneworld. We formulated Maxwell's equations for the case of slowly rotating magnetized



**Fig. 4** The behavior of the radial electric field eigenfunction  $f_1$  for the different values of the brane tension (solid line corresponds to  $q = 0$ , dotted to  $q = -1$ , dashed to  $q = -2$ , dashdotted to  $q = -3$ ).



**Fig. 5** The ratio  $\frac{L_{em\ q \neq 0}}{L_{em\ q=0}}$  as a function of  $q = Q^*/M^2$ .

compact star with non-zero brane tension and considered the particular case of monopolar magnetic field. Despite this configuration may not be considered as realistic it helped us to see the strong influence of brane tension on the electric field of the gravitating object. As the analytical solution is always more valuable for further calculations we attempted to find analytical solution for the dipolar magnetic field configuration. We have derived an approximate analytical expression for the magnetic field just near the surface of the star as a solution of II type hypergeometric equation. This region of the magnetosphere is very important because exactly in this region the processes of plasma generation responsible for the radio emission take place.

We have got equations for the electric field of the slowly rotating magnetized neutron star on branes and solved one of them numerically for different values of brane tension. It is shown that the effect of brane tension on the electromagnetic field of the star is non-negligible (may have the order of tens percents of the initial value) and may help in future in testing the braneworld model.

As an important application of the obtained results we have calculated energy losses of slowly rotating magnetized neutron star in the braneworld and found that the star with non-zero brane parameter will lose more energy than typical rotating neutron star in general relativity. The obtained dependence may be combined with the astrophysical data on pulsar period slowdowns and be useful in further investigations of the possible detection/estimation of the brane parameter.

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